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New Linear - Exponential Distribution

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Abstract :

The proposed distribution is a continuous probability distribution with a single parameter. We named it 'New Linear-exponential distribution'. We have been discussed about probability density function, probability distribution function and moment generating function. Moments about origin and hence, the first four moments about the mean of the proposed distribution have been obtained. Estimation of parameters have been discussed by the method of moments and that of the method of maximum likelihood. To test validity of the theoretical work, goodness of fit has been applied to some data-sets which were used earlier by others. It has been observed that the proposed distribution gives better fit to the most of data-sets than Lindley distribution and One-parameter Linear-exponential distribution.

Keywords : One-Parameter Linear-Exponential distribution, Lindley distribution, Parameters, Moments, Goodness of fit, Estimation.

1. Introduction :

Here, we introduce a single-parameter continuous distribution which is based on the product of a new linear function, $(\pi + x)$, and an exponential function, $e^{-\phi x}$, and hence it is named as 'New Linear-Exponential Distribution (NLED). The nice feature of introducing NLED is that it stands with a single parameter which is to be proposed as to give a better fit to the survival time data of smaller span of time. In the last two decades, many researchers have published different types of Quasi-Lindley distributions having two or more than two parameters which are generalised form of Lindley distribution (1958)[2]. Lindley introduced a continuous probability distribution having a single parameter, known as Lindley distribution (LD), given by its probability density function

$$f_1(x) = \frac{\phi^2(1+x)e^{-\phi x}}{(1+\phi)}\ ; x > 0, \phi > 0 \tag{1}$$

It has been observed that LD (1) gives better fit to the data sets having moderate degrees of variation. When there exist low or high degree of variations in the data-sets, this distribution does not fit the data in better way. Sah (2015) obtained a continuous probability distribution having a single parameter which was named as Mishra distribution (MD), given by its probability density function

$$f_2(x\,;\phi) = \frac{\phi^3}{(\phi^2+\phi+2)}(1+x+x^2)e^{-\phi x}\ ; x > 0, \phi > 0 \tag{2}$$

Mishra distribution of Sah (2015)[4] which gives better fit to the same nature of data-sets than LD[2]. Sah described importance of Poisson-Mishra distribution (2017)[5] and Generalised Poisson-Mishra distribution (2018)[6] in accident proneness. Currently, Sah has obtained Quadratic-Exponential distribution, QED, (2022)[8] and it has been observed that the proposed distribution gives better fit than QED.

The proposed distribution has been constructed on the basic concept of One-Parameter Linear-Exponential Distribution (OPLED) of Sah (2021)[7], given by its probability density function

$$f_3(x\,;\phi) = \frac{\phi^2}{(1+\phi^3)}(\phi^2+x)e^{-\phi x}\ ; x > 0, \phi > 0 \tag{3}$$

The expression (3) was constructed on the basis of a linear function $(\phi^2 + x)$ and an exponential function $(e^{-\phi x})$. In the proposed distribution, we use a new linear function $(\pi + x)$ and exponential function $(e^{-\phi x})$ with a single parameter 'ϕ' and hence, it is named as 'New Linear-exponential distribution (NLED)'. It may be called π distribution.

Probability density function, probability distribution function, moment generating function, moments about origin as well as moments about the mean of the proposed distribution have been obtained. The hazard rate function and the mean residual life function of the proposed distribution have been discussed. Estimation of parameters have been discussed by the method of moments as well as the maximum likelihood methods. The proposed distribution has been fitted to some well-known data-sets which were earlier used by others and it is expected to give better alternative to the same nature of data-sets, having a low degree of variation, than LD, MD and OPLED.

2. Material and Methods :

The present study is based on the theoretical concept of continuous probability distribution. Probability density function and probability distribution function of NLED have been constructed so that it follows all the basic properties of probability distribution. Estimate of the parameter has been discussed by the methods of moments as well as the maximum likelihood methods. Probability of the variable under study for each interval has been calculated. Goodness of fit has been applied to some data-sets which were earlier used by others to test validity of the proposed theoretical work.

3. Results :

The results obtained of the proposed distribution are classified under the following sub-headings.

3.1 New Linear-Exponential Distribution (NLED) and Related Measures

3.2 Statistical Moments and Related Measures of NLED

3.3 The Reliability Function (RF), Hazard Rate Function (HRF) and Mean Residual Life Function (MRLF) of NLED

3.4 Estimation of Parameter of NLED, and

3.5 Goodness of Fit.

3.1 New Linear-Exponential Distribution (NLED) :

In this section, we have described probability density function (pdf), cumulative distribution function (cdf), moment generating function (M.G.F.) and mode (M_0) of NLED. Let X follows NLED with parameter 'ϕ 'such that $x > 0$ and $\phi > 0$. The proposed distribution, NLED, is defined by its probability density function

$$f(x\,;\phi) = \frac{\phi^2}{(1+\pi\phi)}(\pi + x)e^{-\phi x} \tag{4}$$

The expression (4) is the probability density function of NLED.

Probability distribution function : It can be obtained as

$$F(x) = P(X \leq x) = \int_0^x f(x)dx = \frac{\phi^2}{(1+\pi\phi)} \int_0^x (\pi + x)e^{-\phi x}\, dx$$

$$= 1 - \frac{(1 + \pi x + \phi x)}{(1+\pi\phi)} e^{-\phi x} \tag{5}$$

The expression (5) is the probability distribution function of NLED (4).

Moment generating function (M.G.F.) : It can be obtained by

$$M_X(t) = E[e^{tx}] = \int_0^\infty e^{tx} f(x)dx = \frac{\phi^2}{(1+\pi\phi)} \int_0^\infty (\pi + x)e^{-(\phi - t)x}\, dx$$

$$= \frac{\phi^2}{(1+\pi\phi)} \frac{[\pi(\phi - t) + 1]}{(\phi - t)^2} \tag{6}$$

The expression (6) is the M.G.F. of NLED (4).

Mode : It is the value of a random variable X of NLED at which $f'(x) = 0$ and $f''(x) < 0$. We can obtain mode of NLED as follows

Differentiate the expression (4) with respect to $x \sim \text{NLED}(\phi)$.

$$\frac{\partial[f(x;\phi)]}{\partial x} = \frac{\phi^2}{(1+\pi\phi)} \frac{\partial[(\pi + x)e^{-\phi x}]}{\partial x} = \frac{\phi^2}{(1+\pi\phi)} (1 - \pi\phi + x\phi)e^{-\phi x} \tag{7}$$

Applying $f'(x) = 0$, we get

$$x = \frac{(1 - \pi x)}{\phi} \tag{8}$$

The expression (8) is the mode of NLED because $f''(x) < 0$ and it will exist if $(1 - \pi x) > 0$.

Graphical representation of probability density function and distribution function of NLED have been given below.

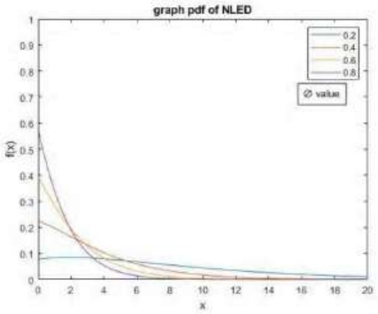

Figure - 1 : Graph of pdf of NLED at $\phi = 0.2, 0.4, 0.6, 0.8$

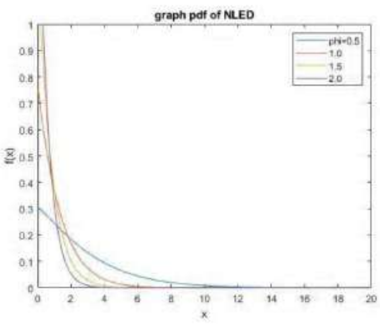

Figure - 2 : Graph of pdf of NLED at $\phi = 0.5, 1.0, 1.5, 2.0$

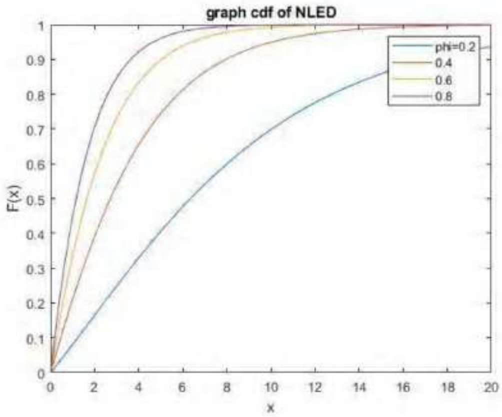

Figure - 3 : Graph of cdf at $\phi = 0.2, 0.4, 0.6, 0.8$

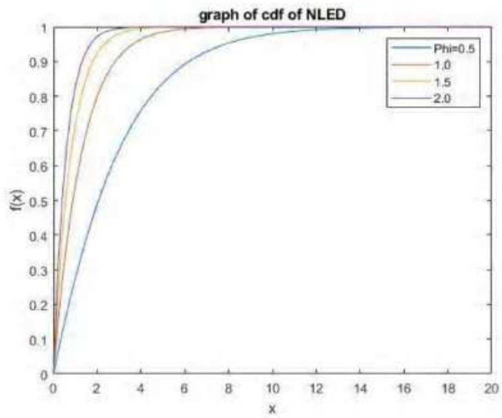

Figure - 4 : Graph of cdf of NLED at $\phi = 0.5, 1.0, 1.5, 2.0$

3.2 Moments and Related Measures of NLED :

In this topic, we have discussed about

- r^{th} moment about origin
- the first four moments about the mean
- over dispersion, shape and size of NLED.

The r^{th} moment about origin : The r^{th} moment about origin is a very useful descriptive measure of Statistics to study about dispersion, shape and size of distribution under study. It can be obtained as

$$\mu_r' = E(X^r) = \int_0^\infty x^r f(x)dx = \frac{\phi^2}{(1+\pi\phi)} \int_0^\infty x^r(\pi + x)e^{-\phi x}\, dx$$

$$= \frac{\phi^2}{(1+\pi\phi)} \frac{\Gamma(r+1)}{\phi^r} \frac{(\pi\phi + r + 1)}{\phi^2} = \frac{r!}{\phi^r} \frac{(1 + r + \pi\phi)}{(1+\pi\phi)} \tag{9}$$

The expression (8) is the general form of the r^{th} moment about origin of NLED (4). It is necessary to obtain statistical moments of the proposed distribution to estimate parameter of NLED (4). Substituting r = 1, 2, 3 and 4 in the expression (9), we get the first four moments about origin of NLED (4) as

$$\mu_1' = \cdot \frac{1!}{\phi} \frac{(2+\pi\phi)}{(1+\pi\phi)} \tag{10}$$

$$\mu_2' = \cdot \frac{2!}{\phi^2} \frac{(3+\pi\phi)}{(1+\pi\phi)} \tag{11}$$

$$\mu_3' = \cdot \frac{3!}{\phi^3} \frac{(4+\pi\phi)}{(1+\pi\phi)} \tag{12}$$

$$\mu_4' = \cdot \frac{4!}{\phi^4} \frac{(5+\pi\phi)}{(1+\pi\phi)} \tag{13}$$

The first four moments about the mean : The first four moments about the mean of the NLED (4) has been obtained as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu'^2$$

$$= \frac{2!}{\phi^2} \frac{(3+\pi\phi)}{(1+\pi\phi)} - \left[\frac{1!}{\phi} \frac{(2+\pi\phi)}{(1+\pi\phi)}\right]^2 = \frac{((\pi\phi)^2 + 4\pi\phi + 2)}{\{\phi(1+\pi\phi)\}^2} = \frac{(2+\pi\phi)^2 - 2}{\{\phi(1+\pi\phi)\}^2} \tag{14}$$

The expression (14) is the variance of the NLED (4). This distribution will be over dispersed under following condition.

$$\mu_2 > \mu_1'$$

Or, $$\frac{(2+\pi\phi)^2 - 2}{\{\phi(1+\pi\phi)\}^2} > \frac{1!}{\phi}\frac{(2+\pi\phi)}{(1+\pi\phi)} \tag{15}$$

Or, $$(2+3\pi\phi)(1-\phi) + \pi\phi(1+\pi\phi-\pi\phi^2) > 0$$

It will be true when $(1-\phi) > 0$ and $(1+\pi\phi-\pi\phi^2) > 0$.

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= \frac{3!}{\phi^3}\frac{(4+\pi\phi)}{(1+\pi\phi)} - 3\left[\frac{2!}{\phi^2}\frac{(3+\pi\phi)}{(1+\pi\phi)}\right]\left[\frac{1!}{\phi}\frac{(2+\pi\phi)}{(1+\pi\phi)}\right] + 2\left[\frac{1!}{\phi}\frac{(2+\pi\phi)}{(1+\pi\phi)}\right]^3$$

$$= \frac{(4+12\pi\phi+12\pi^2\phi^2+2\pi^3\phi^3)}{\{\phi(1+\pi\phi)\}^3} > 0 \tag{16}$$

The expression (16) is the third moment about the mean of NLED (4) which is a positive quantity. Hence, NLED (4) is positively skewed. The fourth moment about the mean of NLED (4) can be obtained as

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= \frac{4!}{\phi^4}\frac{(5+\pi\phi)}{(1+\pi\phi)} - 4\left[\frac{3!}{\phi^3}\frac{(4+\pi\phi)}{(1+\pi\phi)}\right]\left[\frac{1!}{\phi}\frac{(2+\pi\phi)}{(1+\pi\phi)}\right] + 6\left[\frac{2!}{\phi^2}\frac{(3+\pi\phi)}{(1+\pi\phi)}\right]\left[\frac{1!}{\phi}\frac{(2+\pi\phi)}{(1+\pi\phi)}\right]^2$$

$$-3\left[\frac{1!}{\phi}\frac{(2+\pi\phi)}{(1+\pi\phi)}\right]^4 = \left[\frac{(24+96\pi\phi+132\pi^2\phi^2+72\pi^3\phi^3+9\pi^4\phi^4)}{\{\phi(\phi^3+1)\}^4}\right] \tag{17}$$

Shape and size of the proposed probability distribution can be studied by obtaining co-efficient of skewness and kurtosis

$$\gamma_1 = \frac{(4+12\pi\phi+12\pi^2\phi^2+2\pi^3\phi^3)}{(2+4\pi\phi+\pi^2\phi^2)^{3/2}} \tag{18}$$

Here, $\gamma_1 > 0$. Hence, the proposed distribution is positively skewed.

And co-efficient of kurtosis can be obtained by using

$$\beta_2 = \frac{3(8 + 32\pi\phi + 44\pi^2\phi^2 + 24\pi^3\phi^3 + 3\pi^4\phi^4)}{(2 + 4\pi\phi + \pi^2\phi^2)^2} \tag{19}$$

Here, $\beta_2 > 3$. Hence, the proposed distribution is Leptokurtic.

3.3 The Reliability Function, Hazard Rate Function and Mean Residual Life Function of NLED :

The Reliability Function :

Reliability of a system is based on the several techniques and equipment used for the system such as original equipment design, the technique of quality control used during production, methods used for field trials and life testing. Let us define a continuous random variable, X, which follows NLED (4) with parameter ϕ, related to lifetime of a system. Let F and f be the cumulative probability distribution function and probability density function of X. Probability that a component of a system will fail on the interval 0 to t can be obtained by

$$F(t) = \int_0^t f(x)dx = 1 - \frac{(1 + \pi\phi + \phi t)}{(1 + \pi\phi)} e^{-\phi t} \tag{20}$$

The complement of distribution function (20) is called reliability function $[R(t)]$ given by

$$R(t) = P(X > t) = \int_t^\infty f(x)dx = 1 - F(t) = \frac{(1 + \pi\phi + \phi t)}{(1 + \pi\phi)} e^{-\phi t} \tag{21}$$

The expression (21) is the reliability function of NLED (4). The component is said to be working properly at time $t = 0$ and no component work forever without failure i.e.

$$R(t = 0) = 1 \quad \text{and} \quad \lim_{t \to 0} R(t) = 0$$

$R(t)$ is a monotone non-increasing function of t. For $t < 0$, the reliability has no meaning.

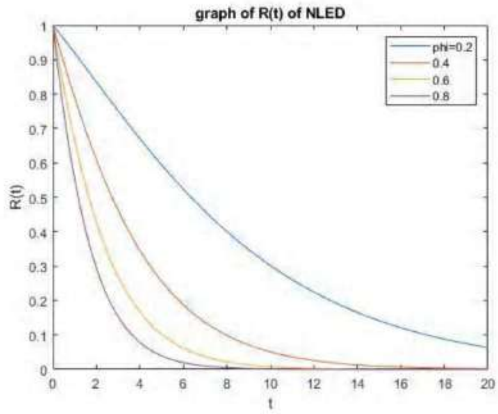

Figure - 5 : Graph of Reliability Function at $\phi = 0.2, 0.4, 0.6, 0.8$

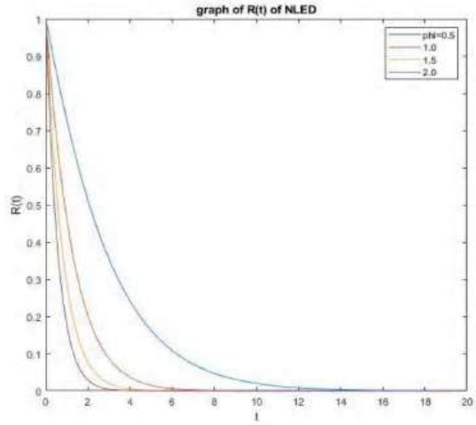

Figure - 6 : Graph of Reliability Function at $\phi = 0.5, 1.0, 1.5, 2.0$

The Hazard Rate Function :

The failure rate function of a system is also called Hazard Rate Function (HRF). Hazard measures the conditional probability of a failure given that the system is working. The failure density (pdf) measures the overall speed of failures. Hazard rate or Instantaneous failure rate measures the dynamic speed of failures.

Let $F(X = t) = P(X \leq t)$ denotes the probability that the system will fail in the interval 0 to t. Let $F[X = (t + \Delta t)] = P[X \leq (t + \Delta t)]$ denotes the probability that the system will fail in the interval 0 to $(t + \Delta t)$. Hence, $F(t + \Delta t) - F(t)$ denotes probability that the system will fail in the interval t to $(t + \Delta t)$. The conditional probability of failure of the system in the interval t to $(t + \Delta t)$ given that the system will survive to time 't' has been obtained as

$$\frac{F(t + \Delta t) - F(t)}{R(t)} \tag{22}$$

The average rate of failure can be obtained by dividing the expression (22) by Δt

$$\frac{F(t + \Delta t) - F(t)}{\Delta t} \cdot \frac{1}{R(t)} \tag{23}$$

Taking the limit $\Delta t \to 0$, we get failure rate of the system given by

$$h(x = t) = \frac{F'(x = t)}{R(t)} = \frac{f(x = t)}{1 - F(x = t)} \tag{24}$$

The main reason for defining the $h(x)$ is that it is more convenient to work with than $f(x)$. Putting the value of $f(x)$ and $1 - F(x)$ of NLED (4) from the expression (4) and (21) in the expression $h(x)$, we get the general expression of the Hazard rate function based on NLED (4) has been obtained as

$$h(x = t) = \frac{\phi^2(\pi + t)}{(1 + \pi\phi + \phi t)} \tag{25}$$

At $\quad t = 0,\ h(x = t = 0) = \dfrac{\pi\phi^2}{(1 + \pi\phi)}$ (26)

It is also obvious that $h(t)$ is an increasing function of t and ϕ.

Mean Residual Life Function :

In reliability studies, the expected additional life time given that a system has survived until time 't' is called mean residual life function. Let a random variable X denotes the life of the system under study, the mean residual life function is given by

$$m(x) = E[X - x / X > x] = \frac{\int_x^\infty [1 - F(t)]dt}{1 - F(x)} \tag{27}$$

Where,

$$1 - F(t) = \frac{(1 + \pi\phi + \phi t)}{(1 + \pi\phi)} e^{-\phi t} \text{ and}$$

$$\int_x^\infty \{1 - F(t)\} dt = \int_x^\infty \frac{(1 + \pi\phi + \phi t)}{(1 + \pi\phi)} e^{-\phi t} dt = \frac{(2 + \pi\phi + \phi x)e^{-\phi x}}{\phi(1 + \pi\phi)} \tag{28}$$

Putting the value of $\int_x^\infty [1 - F(t)]dt$ and $1 - F(x)$ in equation (28), the mean residual life function has been obtained as

$$m(x) = \frac{(2 + \pi\phi + \phi x)}{\phi(1 + \pi\phi + \phi x)} \tag{29}$$

At $\quad x = 0,\ m(x = 0) = \dfrac{(2 + \phi^2)}{\phi(1 + \phi^3)} = \mu_1' \qquad (30)$

It can also be seen that at $x = 0$, the mean residual life function (30) is the mean of NLED (4). It can also be seen that $m(x)$ is a decreasing function of x.

3.4 Estimation of Parameter of NLED :

Estimation is an essential step for modeling of statistical data. The parameter, ϕ, of this distribution has been obtained by using (a) Method of moments and (b) Method of maximum likelihood.

(a) Method of moments : We need the first moment about origin to get estimate of the parameter ϕ. So, the population mean is replaced by respective sample moment and using the expression (10), we can obtain estimate of ϕ as follows.

$$\mu_1' = \cdot \frac{1!}{\phi} \frac{(2 + \pi\phi)}{(1 + \pi\phi)}$$

Or, $\quad \mu_1'\phi + \mu_1'\pi\phi^2 = 2 + \pi\phi$

Or, $(\pi\mu_1')\phi^2 + (\mu_1' - \pi)\phi - 2 = 0$ (31)

The expression (31) is a quadratic equation. An estimate of ϕ can be obtained by using

$$\hat{\phi} = \frac{(\pi - \mu_1') \pm \sqrt{\{(\mu_1' - \pi)^2 + 8\pi\mu_1'\}}}{2\pi\mu_1'} \tag{32}$$

By using the expression (32), we can obtain the estimated value of ϕ.

(b) The method of maximum likelihood : Let $(x_1, x_2, ..., x_n)$ be a random sample of size n from NLED (4). The likelihood function, L, of the NLED (4) is obtained as

$$L = \prod_{i=1}^{n} f(x\,;\phi) = \left(\frac{\phi^2}{1+\pi\phi}\right)^n \left[\prod_{i=1}^{n} (\pi + x_i)\right] e^{-n\phi\bar{x}} \tag{33}$$

and so, the log likelihood function is obtained as

$$\ln L = n \ln \phi^2 - n\ln(1+\pi\phi) + \sum_{x=1}^{n} \ln(\pi + x_i) - n\phi\bar{x} \tag{34}$$

The log likelihood equation is thus obtained as

$$\frac{\partial \ln L}{\partial \phi} = \frac{2n}{\phi} - \frac{n\pi}{(1+\pi\phi)} - n\bar{x} = 0 \tag{35}$$

Or, $$\bar{x} = \frac{2(1+\pi\phi) - \pi\phi}{\phi(1+\pi\phi)} = \frac{(2+\pi\phi)}{\phi(1+\pi\phi)} \tag{36}$$

Where, $\bar{x}$ is the sample mean and it is an unbiased estimate of the population mean. Putting the value of the sample mean and Pie in the expression (36), the estimated value of ϕ is obtained.

3.5 Goodness of Fit and discussion :

The NLED (4) has been proposed for statistical modeling of survival time data having the variance greater than mean. It has been fitted to a number of data-sets to which earlier the Lindley distribution have been fitted by others and to some data-sets this distribution provides closer fits than the LD[2] and OPLED[7]. The fittings of the NLED (4) to the two such data-sets have been presented in the following tables. The first data-set is regarding the survival times (in days) of guinea pigs infected with virulent tubercle bacilli, reported by Bjerkedal (1960)[1], and the second data-set is regarding mortality grouped data for blackbirds species, reported by Paranjpe and Rajarshi (1986)[3]. The expected frequencies according to the Lindley distribution and OPLED (3) have also been given for ready comparison with those obtained by the NLED.

Table - 1 : Survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli

Survival Time (in days)	Observed frequency	Expected Frequency		
		Lindley	OPLED	NLED
0-80	8	16.1	15.9	16.4
80-160	30	21.9	22.0	21.6
160-240	18	15.4	15.5	15.3
240-320	8	9.0	9.0	9.0
320-400	4	5.5	4.8	4.9
400-480	3	1.8	2.5	2.5
480-560	1	2.3	2.3	2.3
Total	72	72.0	72.0	72.0
	$\mu_1' = 181.11111$ $\mu_2' = 43911.11111$	$\hat{\phi} = 0.011$ $\chi^2(df) = 7.77(3)$	$\hat{\phi} = 0.0110429$ $\chi^2(df) = 7.61(3)$	$\hat{\phi} = 0.010860$ $\chi^2(df) = 8.43(3)$

From table (1), we can observe that the value of Chi-square of NLED (4) is slightly greater than the Lindley distribution with same degrees of freedom. From below table (2), we can observe that the value of Chi-square of NLED (4) is less than LD[2] and OPLED[7] with greater degrees of freedom. Hence, it may conclude that in some of the cases NLED (4) gives better fit to the same nature of data-sets, having variance greater than the mean, than LD[2] and OPLED[7].

Table - 2 : Mortality grouped data for blackbirds species reported by Paranjpe and Rajarshi (1986)

Survival Time (in days)	Observed frequency	Expected Frequency		
		Lindley	OPLED	NLED
0-1	192	173.5	151.1	160.8
1-2	60	98.6	99.2	91.0
2-3	50	46.5	53.5	49.0
3-4	20	20.1	26.9	25.5
4-5	12	8.1	11.36	13.0
5-6	7	3.2	5.5	6.5
6-7	6	1.4	2.4	3.2
7-8	3	0.4	1.1	1.6
>8	2	0.3	0.8	1.4
Total	352	352.0	352.0	352.0
	$\mu_1' = 1.568181$ $\mu_2' = 5.005682$	$\hat{\phi} = 0.984$ $\chi^2(df) = 49.85(4)$	$\hat{\phi} = 0.970723$ $\chi^2(df) = 35.67(5)$	$\hat{\phi} = 0.816541737$ $\chi^2(df) = 20.10(5)$

4. Conclusion :

In this paper, we propose a new continuous distribution for statistical modeling of survival time data which is named as New Linear-Exponential distribution (NLED). Several structural properties such as probability density function, probability distribution function, moment generating function, moments about origin as well as the mean have been obtained. The reliability function, hazard rate function and mean residual life function have been discussed. The methods of estimation of parameter have been discussed. Finally, the proposed distribution has been fitted to some data-sets, having variance greater than mean, to test its goodness of fit and it is expected that the NLED (4) gives better fit to some similar nature of the data-sets, having variance greater than the mean, than the LD[2] and OPLED[7].

Conflict of Interest :

The author declared that there is no conflict of interest.

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